NAG Toolbox for MATLAB

f07br

1 Purpose

f07br computes the LU factorization of a complex m by n band matrix.

2 Syntax

```
[ab, ipiv, info] = f07br(m, kl, ku, ab, 'n', n)
```

3 Description

f07br forms the LU factorization of a complex m by n band matrix A using partial pivoting, with row interchanges. Usually $m = n$, and then, if A has k_l nonzero subdiagonals and k_u nonzero superdiagonals, the factorization has the form $A = PLU$, where P is a permutation matrix, L is a lower triangular matrix with unit diagonal elements and at most k_l nonzero elements in each column, and U is an upper triangular band matrix with $k_l + k_u$ superdiagonals.

Note that L is not a band matrix, but the nonzero elements of L can be stored in the same space as the subdiagonal elements of A. U is a band matrix but with k_l additional superdiagonals compared with A. These additional superdiagonals are created by the row interchanges.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **m – int32 scalar**

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2: **kl – int32 scalar**

k_l, the number of subdiagonals within the band of the matrix A.

Constraint: $\mathbf{kl} \geq 0$.

3: **ku – int32 scalar**

k_u, the number of superdiagonals within the band of the matrix A.

Constraint: $\mathbf{ku} \geq 0$.

4: **ab(ldab,$*$) – complex array**

The first dimension of the array **ab** must be at least $2 \times \mathbf{kl} + \mathbf{ku} + 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n coefficient matrix A.

The matrix is stored in rows $k_l + 1$ to $2k_l + k_u + 1$; the first k_l rows need not be set, more precisely, the element A_{ij} must be stored in

A_{ij} $\mathbf{ab}(k_l + k_u + 1 + i - j, j)$ for $\max(1, j - k_u) \leq i \leq \min(m, j + k_l)$.

See Section 8 for further details.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **ab**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldab

5.4 Output Parameters

1: **ab(ldab,$*$) – complex array**

The first dimension of the array **ab** must be at least $2 \times \mathbf{kl} + \mathbf{ku} + 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **info** ≥ 0, **ab** contains details of the factorization.

The upper triangular band matrix U, with $k_l + k_u$ superdiagonals, is stored in rows 1 to $k_l + k_u + 1$ of the array, and the multipliers used to form the matrix L are stored in rows $k_l + k_u + 2$ to $2k_l + k_u + 1$.

2: **ipiv($*$) – int32 array**

Note: the dimension of the array **ipiv** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

The pivot indices. Row i of the matrix A was interchanged with row $\mathbf{ipiv}(i)$, for $i = 1, 2, \ldots, \min(m, n)$.

3: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **kl**, 4: **ku**, 5: **ab**, 6: **ldab**, 7: **ipiv**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, $U(i,i)$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

The computed factors L and U are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(k)\epsilon P|L||U|,$$

$c(k)$ is a modest linear function of $k = k_l + k_u + 1$, and ϵ is the ***machine precision***. This assumes $k \ll \min(m, n)$.

8 Further Comments

The total number of real floating-point operations varies between approximately $8nk_l(k_u + 1)$ and $8nk_l(k_l + k_u + 1)$, depending on the interchanges, assuming $m = n \gg k_l$ and $n \gg k_u$.

A call to f07br may be followed by calls to the functions:

f07bs to solve $AX = B$, $A^{\mathrm{T}}X = B$ or $A^{\mathrm{H}}X = B$;

f07bu to estimate the condition number of A.

The real analogue of this function is f07bd.

9 Example

```
m = int32(4);
kl = int32(1);
ku = int32(2);
ab = [complex(0, 0), complex(0, 0), complex(0, 0), complex(0, 0);
      complex(0, 0), complex(0, 0), complex(0.97, -2.84), complex(0.59, -
0.48);
         complex(0,  0),  complex(-2.05,  -0.85),  complex(-3.99,  +4.01),
complex(3.33, -1.04);
     complex(-1.65, +2.26), complex(-1.48, -1.75), complex(-1.06, +1.94),
complex(-0.46, -1.72);
        complex(0,  +6.3),  complex(-0.77,  +2.83),  complex(4.48,  -1.09),
complex(0, 0)];
[abOut, ipiv, info] = f07br(m, kl, ku, ab)
```

```
abOut =
        0                   0                   0                0.5900 -
0.4800i
        0                   0              -3.9900 + 4.0100i   3.3300 -
1.0400i
        0              -1.4800 - 1.7500i   -1.0600 + 1.9400i   -1.7692 -
1.8587i
        0 + 6.3000i   -0.7700 + 2.8300i    4.9303 - 3.0086i    0.4338 +
0.1233i
   0.3587 + 0.2619i   0.2314 + 0.6358i   0.7604 + 0.2429i       0
ipiv =
          2
          3
          3
          4
info =
          0
```